MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 2, 2016/2017

BDE2014 – DIFFERENTIAL EQUATIONS

(All sections / Groups)

07 MARCH 2017
2.30 p.m – 4.30 p.m
(2 Hours)

INSTRUCTIONS TO STUDENT

1. This Question paper consists of **5 pages** including cover page and mathematical formulas with **4 Questions** only.

2. Attempt **ALL** questions and write your answers in the Answer Booklet provided.

3. The candidate is allowed to use scientific calculators that are permitted to be used in the examination.

QUESTION 1 (25 MARKS)

(a) Solve the *non-exact* equation:

$$(2y^2 + 2y + 4x^2)dx + (2xy + x)dy = 0$$

[8 Marks]

(b) Solve the *Homogeneous* equation

$$(3x^2 - y^2)dx + (xy - x^3y^{-1})dy = 0$$

[8 Marks]

(c) Solve the *Bernoulli* equation

$$\frac{dy}{dx} - y = e^{2x}y^3$$

[9 Marks]

QUESTION 2 (25 MARKS)

(a) Use the *method of undetermined coefficients* to find a particular solution for the following non-homogeneous differential equation:

$$y''(x) + y(x) = 4x \cos x$$

[12 Marks]

(b) Find a general solution for the following differential equation using the *method of variation of parameters*:

$$y'' - 6y' + 9y = t^{-3}e^{3t}$$

[13 Marks]

QUESTION 3 (25 MARKS)

(a) Find a general solution to the given *Cauchy-Euler equation* for $t > 0$:

$$t^2\frac{d^2z}{dt^2} + 5t\frac{dz}{dt} + 4z = 0$$

[8 Marks]

CONTINUED...

(b) Find a second linearly independent solution using *reduction of order*:

$$t^2y'' + 6ty' + 6y = 0 \ , \quad t > 0 \ ; \quad f(t) = t^{-2}$$

where f is a non-trivial solution. **[9 Marks]**

(c) Use the *elimination method* to find a solution for the following linear system:

$$\frac{dx}{dt} = 2x + y - e^{2t} \ ; \quad x(0) = 1 \ ,$$
$$\frac{dy}{dt} = x + 2y \ ; \quad y(0) = -1$$

[8 Marks]

QUESTION 4 (25 MARKS)

(a) Form a partial differential equation by *eliminating the arbitrary constants*:

$$z = ax + by + cxy$$

[7 Marks]

(b) Obtain the partial differential equation by *eliminating the arbitrary function f* from the relation

$$x + y + z = f(x^2 + y^2 + z^2)$$

[7 Marks]

(c) Solve the partial differential equation

$$\frac{\partial^3 y}{\partial x^2 \partial y} = \cos(2x + y)$$

[9 Marks]

(d) Name two types of *elliptic* second order partial differential equation.

[2 Marks]

END OF PAGE

FORMULAS

1) First order DE

Separable Equations: $dy/dx = g(x)p(y)$. Separate the variables and integrate.

Linear Equations: $dy/dx + P(x)y = Q(x)$. The integrating factor $\mu = \exp\left[\int P(x)\,dx\right]$ reduces the equation to $d(\mu y)/dx = \mu Q$, so that $\mu y = \int \mu Q\,dx + C$.

Exact Equations: $dF(x, y) = 0$. Solutions are given implicitly by $F(x, y) = C$. If $\partial M/\partial y = \partial N/\partial x$, then $M\,dx + N\,dy = 0$ is exact and F is given by

$$F = \int M\,dx + g(y)\ , \quad \text{where} \quad g'(y) = N - \frac{\partial}{\partial y}\int M\,dx$$

or

$$F = \int N\,dy + h(x)\ , \quad \text{where} \quad h'(x) = M - \frac{\partial}{\partial x}\int N\,dy\ .$$

When an equation is not separable, linear, or exact, it may be possible to find an integrating factor or perform a substitution that will enable us to solve the equation.

Special Integrating Factors: $\mu M\,dx + \mu N\,dy = 0$ **is exact.** If $(\partial M/\partial y - \partial N/\partial x)/N$ depends only on x, then

$$\mu(x) = \exp\left[\int\left(\frac{\partial M/\partial y - \partial N/\partial x}{N}\right)dx\right]$$

is an integrating factor. If $(\partial N/\partial x - \partial M/\partial y)/M$ depends only on y, then

$$\mu(y) = \exp\left[\int\left(\frac{\partial N/\partial x - \partial M/\partial y}{M}\right)dy\right]$$

is an integrating factor.

Homogeneous Equations: $dy/dx = G(y/x)$. Let $v = y/x$. Then $dy/dx = v + x(dv/dx)$, and the transformed equation in the variables v and x is separable.

Equations of the Form: $dy/dx = G(ax + by)$. Let $z = ax + by$. Then $dz/dx = a + b(dy/dx)$, and the transformed equation in the variables z and x is separable.

Bernoulli Equations: $dy/dx + P(x)y = Q(x)y^n$. For $n \neq 0$ or 1, let $v = y^{1-n}$. Then $dv/dx = (1 - n)y^{-n}(dy/dx)$, and the transformed equation in the variables v and x is linear.

2) Second Order DE

$$ar^2 + br + c = 0\ , \qquad a \neq 0\ .$$

(a) When $b^2 - 4ac > 0$, the auxiliary equation has two distinct real roots r_1 and r_2 and a general solution is

$$y(t) = c_1 e^{r_1 t} + c_2 e^{r_2 t}\ .$$

(b) When $b^2 - 4ac = 0$, the auxiliary equation has a repeated real root $r = r_1 = r_2$ and a general solution is

$$y(t) = c_1 e^{rt} + c_2 t e^{rt} .$$

(c) When $b^2 - 4ac < 0$, the auxiliary equation has complex conjugate roots $r = \alpha \pm i\beta$ and a general solution is

$$y(t) = c_1 e^{\alpha t} \cos \beta t + c_2 e^{\alpha t} \sin \beta t .$$

Method of Undetermined Coefficients

To find a particular solution to the differential equation

$$ay'' + by' + cy = Ct^m e^{rt} ,$$

where m is a nonnegative integer, use the form

(14) $$y_p(t) = t^s (A_m t^m + \cdots + A_1 t + A_0) e^{rt} ,$$

with

(i) $s = 0$ if r is not a root of the associated auxiliary equation;
(ii) $s = 1$ if r is a simple root of the associated auxiliary equation; and
(iii) $s = 2$ if r is a double root of the associated auxiliary equation.

To find a particular solution to the differential equation

$$ay'' + by' + cy = \begin{cases} Ct^m e^{\alpha t} \cos \beta t \\ \text{or} \\ Ct^m e^{\alpha t} \sin \beta t \end{cases}$$

for $\beta \neq 0$, use the form

(15) $$y_p(t) = t^s (A_m t^m + \cdots + A_1 t + A_0) e^{\alpha t} \cos \beta t + t^s (B_m t^m + \cdots + B_1 t + B_0) e^{\alpha t} \sin \beta t ,$$

with

(iv) $s = 0$ if $\alpha + i\beta$ is not a root of the associated auxiliary equation; and
(v) $s = 1$ if $\alpha + i\beta$ is a root of the associated auxiliary equation.

Variation of Parameters

Theorem 7. If y_1 and y_2 are two linearly independent solutions to the homogeneous equation (10) on an interval I where $p(t)$, $q(t)$, and $g(t)$ are continuous, then a particular solution to (11) is given by $y_p = v_1 y_1 + v_2 y_2$, where v_1 and v_2 are determined up to a constant by the pair of equations

$$y_1 v_1' + y_2 v_2' = 0 ,$$
$$y_1' v_1' + y_2' v_2' = g ,$$

which have the solution

(12) $$v_1(t) = \int \frac{-g(t)\, y_2(t)}{W[y_1, y_2](t)}\, dt , \qquad v_2(t) = \int \frac{g(t) y_1(t)}{W[y_1, y_2](t)}\, dt .$$

Note the formulation (12) presumes that the differential equation has been put into standard form [that is, divided by $a_2(t)$].